Abstract

Solar surveillance missions that provide ample warning time of impending solar storms to satellite users in earth orbit are naturally designed to allow the weather forecasting spacecraft to wander as far away from the earth as possible, in the direction of the sun. Vehicles positioned on the \( L_1 \) halo orbits do not provide more than an hour of warning time due to the fact that the \( L_1 \) equilibrium point of the sun-earth system is located at some 1.5 million km from the earth. In order to increase this warning time by a factor of two or more, use is made in this paper of distant retrograde orbits which allow compact sensorcraft to remain in the vicinity of the earth but at substantially larger distances from it than the \( L_1 \) point. However, the sensorcraft transit periodically and only for a limited time inside the surveillance zone centered on the sun-earth axis, such that a certain number of such sensorcraft is needed for continuous surveillance capability. An alternative scheme produces a finite number of passes through the surveillance zone with some transits reaching much further out toward the sun, over several years. Another scheme utilizes a series of miniature probes released from a multi-probe carrier vehicle from a parking halo orbit, to travel on that halo orbit invariant unstable manifold in successive single passes through the surveillance zone, requiring continuing replenishment. This paper shows several sensorcraft and sensor probes release strategies and their associated trajectories for both the distant retrograde orbit and the halo orbit cases. It also provides an estimate of the minimum number of probes needed for continuous coverage for a given time span.

I. Introduction

It is well known that high velocity plasma structures called coronal mass ejections (CMEs) emanating from the sun can interact with the magnetic field of the earth and trigger geomagnetic storms that can affect adversely power grids, communications and spacecraft in earth orbit. Initial studies focused on the use of halo orbits around the sun-earth \( L_1 \) inner libration point using micro satellites equipped with a wide angle imager to track the coronal ejections, and a magnetometer to determine the magnetic polarity of the solar wind. This polarity must have the potential to interact with the magnetic field of the earth to trigger an alarm and alert government and utilities agencies of an impending

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geomagnetic storm. Although the small size and low cost of these small satellites enable affordable periodic replenishment and continuous presence at L₁, the warning time is limited to about 45 minutes due to the L₁ point being distant by some 1.5 million km from the earth, and the solar wind velocity of about 500 km/s. In order to increase this warning time by a factor of two or more, and provide ample time for the confirmation of the impending threat and study the CME characteristics, the spacecraft must wander beyond the L₁ point with at least one satellite transiting through the surveillance zone centered on the sun-earth line of sight at a given time for continuous coverage and monitoring of the solar weather. To this end, two different architectures are examined and analyzed in this paper. The first architecture makes use of the distant retrograde orbits (DROs) which stay in the vicinity of the earth while crossing the earth-sun line periodically at larger distances than the L₁ point. The second architecture uses halo orbits from which a string of satellites is released at regular intervals of time such that they are allowed to travel on the halo orbit unstable manifold which branches out of the L₁ location towards the sun in a three-dimensional twisting path that resides for a certain time within the solar surveillance zone before wandering out of the zone in its heliocentric orbit. In the first architecture, series of satellites is released from a GTO-type orbit at regular times, after some maneuvering, on a path that mimics a multi-period DRO such that these satellites transit through the zone of interest in succession for a continuous monitoring of the CMEs. Section II discusses this architecture. Drawing on the pioneering work of Farquhar and Richardson, regularized nonsingular variables are used to describe the DRO type trajectories, and the transfer trajectories that originate from earth orbit. The libration point orbit dynamics have benefited from the contributions in Refs. 8, 9 and 10 especially as related to the geometry of the stable and unstable manifolds which enable zero cost transfers between certain orbits. The asymptotic departure from halo orbits with zero cost is adopted by the second architecture deployment strategy shown in Section III. The investigation and use of the DRO and halo orbits are depicted in Refs. 11 through 15 in a thorough way.

II. Populating a Certain Distant Retrograde Orbit (DRO) with Small Satellites

Drawing on the considerable contributions of Hénon, Ocampo and Ocampo and Rosborough have suggested the use of Distant Retrograde Orbits for a constellation of satellites to monitor the interplanetary medium at distances larger than the 1.5 x 10⁶ km of the L₁ point in order to increase the warning times of impending geomagnetic storms. The restricted circular three-body model as well as Hill’s approximation or simplified model are used to study via the Poincaré map the characteristics of the DRO orbits in the sun-earth-moon system where the earth and the moon are considered as a single body with mass m₂ such that \( \mu = m_2 / (m_1 + m_2) \) with m₁ the sun’s mass. In a rotating reference system centered at m₂ with x along the m₁ – m₂ direction, y in the ecliptic plane, and z along the out-of-ecliptic direction, and the usual normalizations which set to unity the sun-earth distance a, the system mean motion n, and the sum m₁ + m₂, the differential equations of the circular model are given by

\[
\begin{align*}
\frac{\dot{x}}{r_3} &= -\frac{(1-\mu)}{r_3^3} \begin{pmatrix} x+1 \\ y \\ z \end{pmatrix} - \frac{\mu}{r_2^2} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} 2\dot{y} \\ -2\dot{x} \\ 0 \end{pmatrix} + \begin{pmatrix} x+1-\mu \\ y \\ 0 \end{pmatrix} \\
\end{align*}
\]

where \( r_1 = [(x+1)^2 + y^2 + z^2]^{1/2} \) and \( r_2 = (x^2 + y^2 + z^2)^{1/2} \) are the m₁-s/c and m₂-s/c distances respectively.

Hill’s model is generated by rescaling the distance unit by \( \mu^{1/3} \) and taking the limit \( \mu \to 0 \)

\[
\begin{align*}
\dot{x} - 2\dot{y} &= 3x - \frac{x}{r^3} \\
\dot{y} + 2\dot{x} &= -\frac{y}{r^3}
\end{align*}
\]
The Jacobi constant $C$ for the restricted circular three-body model and the Hill model respectively are given by

$$\frac{1}{2}[(x+1-\mu)^2+y^2] \cdot \frac{1}{r_1} + \frac{\mu}{r_2}$$

and

$$\Omega = \frac{3}{2} x^2 + \frac{1}{r}$$

with $r = (x^2 + y^2 + z^2)^{1/2}$. Various classes of DRO orbits can be investigated by first generating Poincaré sections in the form of the $(x, \dot{x})$ plane for the planar problem of motion in the ecliptic plane ($y = 0$), for various values of the energy constant $C$. Considering the $\dot{y} < 0$ crossings through the Poincaré section $\Sigma$, retrograde orbits about the origin, or the earth, are generated using Hill’s model. It is known that a periodic orbit exists at the center of the bounded curves of the Poincaré section plots. Thus a family of single periodic DRO’s, stable in the linear sense are obtained as well as n-period unstable retrograde orbits which do not feature any close encounters with the earth. Unstable periodic orbits with repeated passes close to the earth are discussed in Ref. 7 and examples of simple period as well as 2 and 4 period orbits are shown. These close encounter orbits are used to inject a spacecraft from low earth orbit, and an insertion maneuver will later capture a DRO-type orbit to carry out its mission.

In the restricted circular three-body model, both stable DRO’s as well as unstable ERO’s (Earth Return Orbits) can be generated. The stable simple period DRO’s are shown to be linearly stable up to $x$ values of 10 million km from the earth. The simple period ERO’s belong to the family of the planar Lyapunov orbits about either the $L_1$ or $L_2$ collinear libration points which are continued and extended towards the earth until a close approach is achieved. Multiple period or n-cycle ERO’s are also shown in Refs. 7 and 14. However these multi-period ERO’s tend to get less wide along the $x$ direction than their simple period counterpart for the same closest distance to the earth. An example using $x_0 = 100,000$ km at closest approach is used in Ref. 7, and a simple period ERO with a period of roughly 400 days, as well as a period 2 and period 4 ERO of period 565 and 974 days respectively, are generated. The simple period ERO wanders beyond the 3 million km mark along $x$ only once in its orbit but the 2 and 4-period ones wander at a maximum $x$ value of $2 \times 10^6$ km twice, and at $1.5 \times 10^6$ km and $2.5 \times 10^6$ km twice each respectively, albeit along longer period orbits. However, the multi-period ERO’s provide a higher frequency of entry into the surveillance zone centered along the $x$ axis, per time unit than their simple-period counterpart, and for this reason they are more interesting as candidate trajectories for inexpensive throw-away small satellites which would be on station more than once in their mission lifetime.

In order to search for distant retrograde orbits with close passage near the earth a computer program developed in Ref. 5 is used. The software uses the restricted circular model but its rotating axes are centered at the $L_1$ libration point with $x$ pointing towards the earth, and $y$ along earth’s velocity vector in its heliocentric orbit. The $z$ axis is along the normal to the ecliptic plane. The barycentric equations of motion are given by

$$\ddot{x} - 2\dot{y} - x = -\frac{(1-\mu)(x+\mu)}{r_1^3} - \frac{\mu(x-1+\mu)}{r_2^3}$$

$$\ddot{y} + 2\dot{x} - y = -\frac{(1-\mu)y}{r_1^3} - \frac{\mu y}{r_2^3}$$

$$\ddot{z} = -\frac{(1-\mu)z}{r_1^3} - \frac{\mu z}{r_2^3}$$

where $\tilde{r}_1 = (x+\mu)\tilde{x} + y\tilde{y} + z\tilde{z}$, $\tilde{r}_2 = (x-1+\mu)\tilde{x} + y\tilde{y} + z\tilde{z}$ are the sun-s/c and earth-s/c vectors with the hatted quantities standing for unit vectors along the three rotating axes. The $L_1$-centered equations are given by

$$\ddot{x} = 2\dot{y} + (1-\gamma_L)x - \left[(1-\mu)/r_1^3\right](1-\gamma_L)x + (\mu/r_2^3)(\gamma_L - x) - \mu$$

where $\gamma_L = \frac{1}{2} x^2 + \frac{1}{r}$.
\[
\ddot{y} = -2\dot{x} + \dot{y} - \left(\frac{1 - \mu}{r_1^3}\right) y - \left(\frac{\mu}{r_2^3}\right) y
\]

(9)

\[
\ddot{z} = -\left(\frac{1 - \mu}{r_1^3}\right) z - \left(\frac{\mu}{r_2^3}\right) z
\]

(10)

where once again \(\mu\) and \((1 - \mu)\) are the dimensionless masses of the earth and the sun and where \(\gamma_L\) is the L1-earth distance.

\(r_1\) and \(r_2\) are now written as

\[
r_{1} = \left[\left(1 - \gamma_L\right) + \bar{x}\right]^2 + \bar{y}^2 + \bar{z}^2 \right]^{1/2} \quad \text{and} \quad r_{2} = \left[\left(\bar{x} - \gamma_L\right)^2 + \bar{y}^2 + \bar{z}^2 \right]^{1/2}.
\]

The coordinates \(\bar{x}, \bar{y}, \bar{z}\) are dimensionless and the derivatives are with respect to non-dimensional time. A regularized form of the L1-centered equations is used for accurate numerical integration\(^5\)

\[
\frac{du_1}{d\tau} = v_1
\]

(11)

\[
\frac{du_2}{d\tau} = v_2
\]

(12)

\[
\frac{du_3}{d\tau} = v_3
\]

(13)

\[
\frac{du_4}{d\tau} = v_4
\]

(14)

\[
\frac{dv_1}{d\tau} = \left(\frac{\hbar}{2}\right) u_1 + 2n_E \left[\left(u_1^2 + u_2^2\right)v_2 + \left(u_2u_3 - u_1u_4\right)v_3 - \left(u_1u_3 + u_2u_4\right)v_4\right] + \frac{1}{2}\left(u_1^2 + u_2^2 + u_3^2 + u_4^2\right)\left(u_1F_1^* + u_2F_2^* + u_3F_3^*\right)
\]

(15)

\[
\frac{dv_2}{d\tau} = \left(\frac{\hbar}{2}\right) u_2 + 2n_E \left[-\left(u_1^2 + u_2^2\right)v_1 + \left(u_2u_4 + u_1u_3\right)v_3 + \left(u_2u_3 - u_1u_4\right)v_4\right] + \frac{1}{2}\left(u_1^2 + u_2^2 + u_3^2 + u_4^2\right)\left(-u_2F_1^* + u_1F_2^* + u_4F_3^*\right)
\]

(16)

\[
\frac{dv_3}{d\tau} = \left(\frac{\hbar}{2}\right) u_3 + 2n_E \left[u_1u_4 - u_2u_3\right)v_1 - \left(u_1u_3 + u_2u_4\right)v_2 + \left(u_3^2 + u_4^2\right)v_4\right] + \frac{1}{2}\left(u_1^2 + u_2^2 + u_3^2 + u_4^2\right)\left(-u_3F_1^* - u_4F_2^* + u_1F_3^*\right)
\]

(17)

\[
\frac{dv_4}{d\tau} = \left(\frac{\hbar}{2}\right) u_4 + 2n_E \left[u_2u_4 + u_1u_3\right)v_1 + \left(u_1u_4 - u_2u_3\right)v_2 - \left(u_3^2 + u_4^2\right)v_3\right] + \frac{1}{2}\left(u_1^2 + u_2^2 + u_3^2 + u_4^2\right)\left(u_4F_1^* - u_3F_2^* + u_2F_3^*\right)
\]

(18)

\(n_E = 1.990986606 \times 10^{-7}\) rad/s is the sun-earth angular rate, \(\tau\) is a fictitious time related to the physical time \(t\) by \(dt = r_2 d\tau\), and the quantity \(\hbar\) is given by

\[
\hbar = n_E^2 \left(\frac{1}{2} \left[\left(x + (1 - \mu) - \gamma_L\right)^2 + y^2\right] + (1 - \mu)/r_i - C_J / 2\right)
\]

(19)

The Jacobi constant \(C_J\) is given by \(C_J = 2U^* - \left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right)\) with the pseudopotential \(U^*\) written as

\[
U^* = \frac{1}{2} \left[\left(x + (1 - \mu) - \gamma_L\right)^2 + y^2\right] + (1 - \mu)/r_1 + \mu/r_2.
\]

The \(u\)-variables are related to the cartesian coordinates
through $x^* = u_1^2 - (u_2^2 + u_3^2 - u_4^2)$, $y = 2u_1u_2 - 2u_3u_4$, $z = 2u_1u_3 + 2u_2u_4$, with $x^* = x - \gamma_L$ and the forcing functions given by

\[ F_1^* = (1 - \gamma_L)n_E^2 + n_E^2x - \left[ (1 - \mu)/r_1^3 \right] (1 - \gamma_L) + x \gamma_L - \mu n_E^2 \tag{20} \]

\[ F_2^* = yn_E^2 - \left[ (1 - \mu)/r_1^3 \right] n_E^2y \tag{21} \]

\[ F_3^* = -\left[ (1 - \mu)/r_1^3 \right] n_E^2z \tag{22} \]

Finally, the physical time is obtained from the integration of $\frac{dt}{d\tau} = u_1^2 + u_2^2 + u_3^2 + u_4^2$. Given initial conditions in the form of $x_0, y_0, z_0, \dot{x}_0, \dot{y}_0, \dot{z}_0$, the corresponding eight quantities $u_1(0), u_2(0), u_3(0), u_4(0), u_1'(0), u_2'(0), u_3'(0), u_4'(0)$ are determined and the simultaneous integration of the equations (11)-(18) as well as $\frac{dt}{d\tau}$ carried out. Backwards integration is carried out with respect to $\tau' = -\tau$ after changing the sign of the second term in each of Eqs. (15)-(18) and after dropping the $n_E$ and $n_E^2$ factors in $F_1^*, F_2^*, F_3^*$ and $n_E^2$ to render all the quantities at hand unitless. Starting from a point in space, the backwards integration is stopped at closest approach to the earth by also iterating on the transfer time $t_f$ such that the closest approach point takes place at time zero.

Restricting the motion to the ecliptic plane, small velocity changes $\Delta\dot{x}$ and $\Delta\dot{y}$ at the initial point are guessed and the backwards integration carried out, and these two search velocity changes searched on until a desired earth flyby distance $h_T$ and argument of perigee $\omega_T$ are matched. All the necessary transformations to the earth-centered equatorial system are shown in detail in Ref. 5 from which the osculating orbit elements are readily obtained.

The geometry of the trajectory and its intersection with the conical surveillance zone is shown in Figure 1. As a function of $x$, the radius of the base of the 15 deg half angle cone is given by $r_{\text{max}} = (R_1 - x) \tan 15^0$ where $R_1$ is the L1-earth distance. The spacecraft is located from L1 and the earth center by $r_L$ and $R_g$, while its distance from the x axis is depicted by $r_{\perp} = (y^2 + z^2)^{1/2}$. A numerical search is next carried out

![Figure 1. Conical surveillance zone and trajectory geometry.](image-url)
starting at point \( x_0 = -3.5 \times 10^6 \) km, \( y_0 = 0 \), \( z_0 = 0 \), \( \dot{x}_0 = 0 \), \( \dot{y}_0 = 350 \text{ m/s} \), \( \dot{z}_0 = 0 \) and searching on the \( \Delta \dot{x} \), \( \Delta \dot{y} \) quantities such that the backwards iterated trajectory achieves a close approach of the earth. A solution using \( \Delta \dot{x} = 0.614847 \text{ km/s} \), \( \Delta \dot{y} = 1.617502 \text{ km/s} \) achieving an \( h_T = 137.616 \) km is found and shown traced forward in time in Figure 2 from the earth to the \( x = -3.5 \times 10^6 \) km point which in effect is equivalent to a distance of 5 million km from the earth itself. Figure 3 shows the evolution of the various quantities, namely \( x \), \( y \), \( r_{\text{max}} \), \( r_{\perp} \), \( R_g \) as a function of time \( t \) during the first 300 days of this trajectory. The

![Figure 2](image1.png)

**Figure 2.** x-y trace of a backwards generated trajectory intersecting the earth.

![Figure 3](image2.png)

**Figure 3.** Evolution of \( x \), \( y \), \( r_{\text{max}} \), \( r_{\perp} \), and \( R_g \) distances during first 300 days from earth launch.
portions of these type of trajectories that are within the surveillance zone must have \( x \) negative and \( r < r_{\text{max}} \) which is clearly the case just before the 300 day mark for a duration of about 20 days. If this trajectory is integrated further in time, it is seen in Figure 4 that another passage in the surveillance zone takes place before the s/c moves farther out from the vicinity of the earth. It will eventually return near the earth after moving completely once around the sun in this rotating frame depiction. This numerical search is pursued further by selecting other \( \omega \) target values leading to a path that closely resembles a multiple-cycle DRO as shown in Figure 5. Here five passages in the surveillance zone are possible before

![Figure 4](image-url)

**Figure 4.** Repeated passages of spacecraft through surveillance zone: \( x-y \) trace.

![Figure 5](image-url)

**Figure 5.** Distant retrograde orbit-type trajectory emanating from earth.
the spacecraft departs farther away from the vicinity of the earth. The initial conditions at departure from the earth at time zero are given by the following elements, namely \( a_0 = -129966.259 \text{ km, } e_0 = 1.043418887, \ i_0 = 23.44 \text{ deg, } \Omega_0 = 0 \text{ deg, } \omega_0 = 319.283459 \text{ deg, } \theta^*_0 = 0 \text{ deg with } \theta^* \text{ standing for the true anomaly.} \) This trajectory is effectively planar and it is contained entirely in the ecliptic plane. Its perigee radius \( r_p \) at departure from the earth at time zero is 5642.990 km which is less than the radius of the earth \( R_\oplus \) which is considered here as a point mass. Minor adjustments in \( a_0 \) and \( e_0 \) will provide an \( r_p > R_\oplus \) as desired while still keeping the overall geometry of Figure 5. In fact changing only \( a_0 \) to \(-139966.259 \text{ km}\) will increase \( r_p \) to 6077.17 km without affecting substantially the geometry of the first four passages through the surveillance zone. In Figure 6, the trajectory of Figure 5 is shown as a function of time with

![Figure 6. Evolution of x, y, r_max, r⊥, and Rg variables of DRO-type trajectory.](image)

the x and y components in blue and green respectively, while \( r_{\text{max}} \) and \( r_{\perp} \) are depicted by the yellow and red curves respectively. The \( R_g \) distance is shown in magenta. The five entries into the surveillance zone are clearly visible in Figure 6. If succeeding satellites trace this path, then a fleet of about twenty such vehicles will suffice to ensure that at least one of them will be transiting through the surveillance zone at a given time for a continuous monitoring of the solar weather upstream of the L1 point. The spacecraft distance and velocity relative to L1 are shown in Figures 7 and 8 with the first two as well as the last entry into the zone taking place at or near the 5 million km mark. Further shaping of the trajectory may lead to further entry periods, and if maneuverable vehicles are used instead, to effective n-cycle periodic DRO’s for even more extended useful lifetimes.
Figure 7. Evolution of $L_1$-relative distance of DRO-type transfer trajectory.

Figure 8. Evolution of $L_1$-relative velocity of DRO-type transfer trajectory.
A Possible Deployment Strategy

When maneuverable vehicles are considered, it is advantageous to deploy several vehicles on a single launcher into an intermediate orbit such as a GTO type of orbit before injecting each individual spacecraft onto the deep space trajectory.

A GTO type orbit inclined at 23.44 deg of say a = 24446 km, e = 0.724 will experience a nodal regression of about 0.45 deg/day and an advance of its perigee of about 0.66 deg/day due to the earth gravity field. If we consider a launch of say five satellites in a cluster onto this GTO orbit, then releasing or injecting each vehicle from this GTO at a 20 day interval, will result in the string of spacecraft spaced at 20 days as desired to repeat the pattern of Figure 6 for continuous presence in the surveillance zone.

However each vehicle must adjust its orbit argument of perigee and node prior to injection. In a crude analysis, the GTO itself could be biased in both $\omega$ and $\Omega$ in such a way that the first and last vehicles would perform equally demanding maneuvers prior to injection, with the in-between vehicles requiring much less $\Delta V$ to perform smaller adjustments in $\omega$ and $\Omega$.

<table>
<thead>
<tr>
<th>s/c</th>
<th>$\Delta \omega$ (deg)</th>
<th>$\Delta \Omega$ (deg)</th>
<th>$\Delta V_N$ (m/s)</th>
<th>$\Delta V_h$ (m/s)</th>
<th>$\Delta V$ total (m/s)</th>
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Table 1 shows an initial bias in $\omega$ of 26 deg. and in $\Omega$ of 18 deg such that with $\omega$ and $\Omega$ drifting in time, the subsequent second and third spacecraft require smaller changes, with the fifth vehicle requiring a similarly larger change as vehicle 1. The $\Delta V$ calculations can be carried out through the use of the vacant focus theory which shows that the in-plane orbit elements obey the following perturbation laws

$$\dot{a} = \frac{2a^2}{\mu} V f_T$$

$$\dot{e} = 2(e + c_{\theta^*}) \frac{f_T}{V} - \frac{r}{a} s_{\theta^*} \frac{f_N}{V}$$

$$\dot{c_{\theta^*}} = 2s_{\theta^*} \frac{f_T}{V} + \left(2e + \frac{r}{a} c_{\theta^*}\right) \frac{f_N}{V}$$

If a change $\Delta \omega$ is desired while constraining $\Delta e = 0$, then from Eq. (24)

$$\Delta e = \frac{2 \Delta V_T}{V} - (c_{\theta^*} + e) \frac{\Delta V_N}{V} \frac{r}{a} s_{\theta^*} = 0$$

or

$$\Delta V_T = \frac{r}{a} s_{\theta^*} \frac{\Delta V_N}{2(c_{\theta^*} + e)}$$
This expression is now used in Eq. (25) written as

\[ e \Delta \omega = \frac{2 \Delta V_T}{V} s_{\theta^\ast} + \frac{\Delta V_N}{V} \left( 2e + \frac{r}{a} c_{\theta^\ast} \right) \]

yielding for \( \theta^\ast = 180^\circ \) and corresponding \( V = \left[ \frac{\mu}{a} \left( \frac{1-e}{1+e} \right) \right]^{1/2} \)

\[ \Delta V_N = \frac{e \Delta \omega}{(1+3e)} \left[ \frac{\mu}{a} \left( \frac{1-e}{1+e} \right) \right]^{1/2} \tag{27} \]

At apogee, \( \Delta V_T = 0 \) from Eq. (26) such that the \( \Delta \) change from Eq. (23) is also equal to zero

\[ \Delta a = \frac{2a^2}{\mu} V \Delta V_T \tag{28} \]

Therefore, \( \Delta V_N \) is the total \( \Delta V \) needed to make the \( \Delta \omega \) change using a single impulse at the apogee of the GTO orbit. This maneuver is more conservative than an optimal two-impulse maneuver but it is sufficient for our first order analysis.

For our GTO orbit, Eq. (27) yields a value at \( \Delta V_N \) of about 6.43 m/s per degree of \( \Delta \omega \) rotation of the line of apsides. The corresponding values are shown in Table 1 for the five vehicles. It is however more expensive to adjust the \( \Omega \) values prior to the injection of each vehicle from the GTO drifting orbit. From the general perturbation equation

\[ \dot{\Omega} = \frac{rf_h}{h_1} s_\theta \tag{29} \]

where \( \theta = \omega + \theta^\ast \), and \( f_h \Delta T = \Delta V_h \) represents the out-of-plane velocity change, we have

\[ \Delta \Omega = \frac{r \Delta V_h}{h_1} s_\theta \tag{30} \]

which for \( \omega = 0 \) for convenience, has a maximum at \( c_{\theta^\ast} = -e \) resulting in an impulse location at \( r = a \). For our GTO orbit, \( \theta^\ast = 136.385 \) deg and

\[ \Delta V_h = 1.606266801 \Delta \Omega \tag{31} \]

or roughly 28 m/s per degree of \( \Omega \) rotation. Because the perigee velocity of the GTO orbit is \( V_p = 10.092 \) km/s, and because the example of Figure 5 requires a perigee velocity at departure of \( V_p^* = 11.184 \) km/s, then an injection \( \Delta V \) from the GTO orbit at perigee of 1.092 km/s is also needed besides the \( \Delta V_N \) and \( \Delta V_h \) maneuvers of Table 1. Adding all three \( \Delta V \)'s to the GTO orbit results in the totals also shown in Table 1.

**III. Deployment from an L1-halo Orbit onto Invariant Unstable Manifolds**

In this scenario, only early CME warnings that are at least twice the warning time afforded by a spacecraft in an L1 halo orbit are considered. Further, useful warnings are assumed possible only when the spacecraft resides within a tube having a radius of 4 sun radii that is centered on the earth-sun line. Using this early warning criterion, what performance can be gained from a “garage”, holding many small spacecraft, that resides in an L1 halo? One possible solution that minimizes the propulsion needs of the small spacecraft in the “garage”, is to boost them out of the “garage” onto invariant unstable manifolds at intervals that will provide earth with continuous early warning.

To begin, a circular restricted three-body problem L1, halo having a maximum z excursion, away from the ecliptic plane, of 400,000 km was chosen for the nominal “garage” orbit. Figure 9 shows the three projections of the orbit in the usual earth centered rotating frame. (Note: Similar to the previous section of
this paper, the mass of the earth and the moon have been combined to compose a single fictitious planet, referred to here as the earth/moon, that is the second primary in this three-body model. The size of the halo has not been optimized, but has been used rather as a representative case; it is large enough to allow uninterrupted communication with earth, but small enough to give reasonable on-station time.

The six eigenvalues of the monodromy matrix associated with this nominal, periodic, \( L_1 \) halo, consist of one stable (< 1), one unstable (> 1), and 4 unity magnitude (= 1) eigenvalues. Accordingly, a two-dimensional invariant stable manifold, in six-dimensional state space, asymptotically approaches the periodic solution as time increases in a positive direction. Similarly, by reversing the flow of time, a two-dimensional invariant unstable manifold, in six-dimensional state space, asymptotically approaches the periodic solution. The unstable invariant manifold can be approximated at any given point on the halo, thus reducing the dimensionality of the manifold to one, by the eigenvector associated with the unstable eigenvalue of the appropriate monodromy matrix (this particular monodromy matrix is found by integrating the state transition matrix from the point of interest over one halo period). This unstable eigenvector is guaranteed to be tangent to the unstable invariant manifold only at the halo orbit. However, a small perturbation, on the order of a couple of hundred kilometers in position, in the direction of the eigenvector will remain close enough to the unstable manifold that integration backwards in time from this perturbed state will still asymptotically approach the halo orbit. Further, this perturbation is large enough to allow one to quickly follow the unstable invariant manifold away from the vicinity of the halo. It is the behavior of the invariant unstable manifold, away from the halo, that will be leveraged in the design of this early CME warning system.

Figure 9: Three planar projections of a 400,000 km \( L_1 \) halo orbit as seen in the earth centered rotating reference frame. The black asterisks represent points on the halo that will be used to approximately locate the nearby invariant unstable manifold.
Figure 10 shows the 400,000 km L$_1$ halo with three representative unstable manifolds emanating from three points on the halo, each depicted with a black asterisk (these same halo departure points are also denoted by black asterisks in Figure 9). The manifolds, themselves, initially spiral about the sun-earth line while progressing ahead of the earth due to a larger effective mean motion. These three manifolds, as seen in Figure 11, are closer to the sun than the “garage” and stay within the prescribed early warning sun-earth tube for almost 290 days. However, they remain at least twice as far from the earth as the L$_1$ point and in the coverage tube for only 38 days. Thus, to guarantee continuous warning times that are two times better than is possible from a L$_1$ halo, a new probe must be launched from the “garage” roughly every 38 days. Launching 10 probes per year would only be feasible for very small inexpensive probes. To this end, one advantage to using the unstable invariant manifold trajectories is that the “garage” could provide the boost necessary to enter the trajectory, thus reducing the propulsion needs (i.e., size, complexity, and cost) of the probes.

Figure 10: Three planar projections of three invariant unstable manifolds emanating from a 400,000 km L$_1$ halo. The black asterisk signifies where on the halo the manifolds originate.

Future considerations are focused at increasing the on-station time from 38 days per spacecraft. This may be accomplished by optimizing the size of the nominal halo and by performing critically placed maneuvers. Of course, there will be a trade-off between requiring fewer, but more sophisticated, spacecraft that can stay on-station longer versus needing more inexpensive small spacecraft to meet the mission objective of providing the earth with continuous early CME warning.
Figure 11: The three manifolds integrated from the 400,000 km halo stay within the 4 Sun Radii
contained cylinder radius for nearly 290 days, but only give better than twice the warning
time of a spacecraft in a L1 halo for 38 days.

IV. Conclusion

Two solar surveillance zone population strategies using a series of small satellites that transit through the
earth-sun line at distances larger than the L1 libration point have been discussed.
The first architecture consists of using an intermediate GTO-type orbit from where the vehicles are
injected into certain distant retrograde orbits at regular intervals of time flying in the ecliptic plane and
crossing the earth-sun line periodically at distances larger than the L1 point for increased warning time of
impending geomagnetic storms.
Because a cluster of vehicles must be flown on a single booster to the GTO orbit, and due to the
precession of that parking orbit between successive releases onto the DRO-type orbits, it is shown that
$\Delta V$ requirements for maneuvering prior to injection onto the ecliptic plane are not excessive in view of
the small mass of each individual spacecraft.
A second architecture that releases each spacecraft at regular intervals of time from a parking halo orbit
onto its corresponding unstable manifold with essentially zero cost in $\Delta V$ has also been discussed and the
three dimensional flight paths depicted after assuming a proper dispersion of the spacecraft from the
carrier vehicle in halo orbit prior to individual release.
For both architectures, the minimum number of vehicles is determined for continuous solar weather
monitoring over a given time span.
References


